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THE DISTRIBUTION OF REFLECTING CROSS SECTIONS OF SATELLITES

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It was determined that the long-term fluctuation
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ABSTRACT

The bistatic radar cross section of five artificial earth satellites was measured with the Navy's Space Surveillance sensors. The statistical variation of these quantities must be known in order to calculate system probability of detection and is also of value in forming estimates of target geometry, size, and motion.

It was determined that the long-term fluctuation in apparent size of these objects follows a lognormal distribution with a standard deviation probably dependent on object geometry and motion. For symmetrical, stabilized objects the standard deviation is approximately 5 db, while for a cylindrical tumbling object it was found to be about 7 db.

Confidence intervals for reflecting cross section are given in terms of deviation from the log mean of the calculated values of area.

PROBLEM STATUS

This is an interim report on one phase of the problem; work is continuing.

THE DISTRIBUTION OF REFLECTING
CROSS SECTIONS OF SATELLITES

INTRODUCTION

Many variables must be considered in determining the probability that a particular target will be detected by the Space Surveillance System. Among these are the target size, shape, and altitude, transmitter power, receiver sensitivity, antenna gains, and the sky temperature. The classical radar equation contains the factor σ, the effective reflecting area of the target, as well as the power, antenna gains, noise figure, etc. This factor σ is the least well known of all. Where antenna gains and noise figures can be measured and controlled, the effective reflecting area is a complex function of target size, shape, and attitude. If target position, power, and antenna gains, are known, it is possible to compute the effective target size.

DISCUSSION

In order to calculate the effective size of a satellite, we must determine position so that distances and antenna gains may be set into the radar power equation. To this end, special recordings were made, using the facilities of the Space Surveillance System standby monitor center at the Naval Research Laboratory. Arrangements were made so that the receiving stations would supply step calibrations after each pass of the selected satellites. The calibration of the automatic gain control channel in 10-db steps allows determination of received power to within one db by careful interpolation. Antenna gains were determined from pattern plots of the various system antennas. The known data – transmitted power, received power, antenna gains, and wavelength – were inserted in the equation

$$\frac{P_R}{P_T} = \frac{G_T \, G_R \, \sigma \, \lambda^2}{(4\pi)^3 \, R_T^2 \, R_R^2}$$

to determine σ. In this equation

P_R = Received power

P_T = Transmitted power

G_T = Transmitter antenna gain

G_R = Receiver antenna gain

σ = Reflecting cross section

λ = Wavelength

R_T = Distance of satellite from transmitter

R_R = Distance of satellite from receiver.

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Since the satellites may be illuminated by both transmitters of the Space Surveillance System, it is necessary to consider the combined contributions. Because of the transmitter antenna directivity, the power delivered to the target at large zenith angles falls off rapidly near the horizon. As a result, low satellites do not receive significant power from the western transmitter during an eastern-complex pass and vice versa. For calculations involving both transmitters it is assumed that the two transmitter contributions would be incoherent, so that the powers are simply added. Antenna patterns and satellite data are assembled in Appendix A along with some sample calculations. It is found that the calculated areas are distributed widely.

Various causes have been suggested for the target scintillation: varying polarization between incoming wave and receiving antenna, varying medium attenuation, irregular target shape, etc. Since the purpose of this study is to determine the probability of detecting a satellite at a given distance, it is enough to determine the distribution of the apparent reflecting cross section.

Several distributions were investigated. A cosine-squared distribution was tried with the thought that a plane wave of random polarization striking an antenna could be the major cause of the variation. There was no significant agreement with the data. Then normal and Rayleigh distributions were tried. The Rayleigh was better but not significantly close. Next a lognormal distribution was tried. Specifically, each calculated area was expressed in decibels above one square foot. These logarithmic values show good agreement to a normal distribution about the mean of the decibel area values. As it happens the rms (standard) deviations from the mean were different for the five satellites studied.

The distributions of calculated areas as determined in Appendix A for five satellites are shown on probability graph paper (Figs. 1-4). A straight line plot on probability paper indicates normal distribution; since the ordinates on Figs. 1-4 are in logarithmic units, a straight line indicates lognormal distribution. On each sheet a straight line is drawn for a theoretical distribution that includes 63% of the population between plus and minus one standard deviation from the mean. The agreement of the data with the lognormal distribution appears good. There is no obvious physical reason for the lognormal distribution, but researches in other lines of work have shown that the lognormal distribution results when many independent variables are multiplied together. The proceedings of a symposium held at the University of California* has papers by two authors who refer to a French doctoral thesis on the lognormal distribution to support their explanation of certain fading phenomena. The original French paper by J. J. Agard was "The lognormal law and its applications to the mining searches." Despite the wide gulf between mining search and radio propagation, the analogy seems to fit. Where many independent variables are multiplied together, the value of the function varies according to the lognormal law. In any case, the apparent close agreement makes certain probability calculations reasonable. Certain statistical tests, as given in Appendix B, were applied to the data to determine significant agreement with the lognormal distribution.

Let us take a hypothetical example of satellite detection. Assume that a signal is just detectable if it equals the receiver noise (plus sky noise, if significant). Place a satellite of known size at such range that according to the radar equation, it is just detectable. From our observation of variation of reflection cross section, we can conclude that we should detect the satellite for one half of the time, i.e., the returned signal will be greater than noise for 50% of the time. In order to make the returned power greater than noise for a greater part of the time we must increase transmitter power or reduce range. If we assume a standard deviation of the cross section as 6 db, then an increase of received

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*"Statistical Methods in Radio Wave Propagation," proceedings of a symposium held June 18-20, 1958, edited by W. C. Hoffman, London, New York:Pergamon Press, 1960.
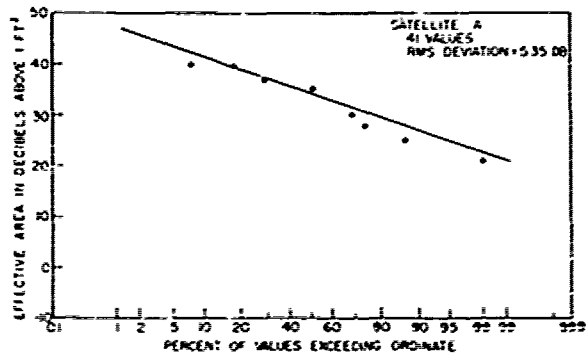
Fig. 1 - Agreement of typical cross sections cal-
culated for satellite A with lognormal distribution
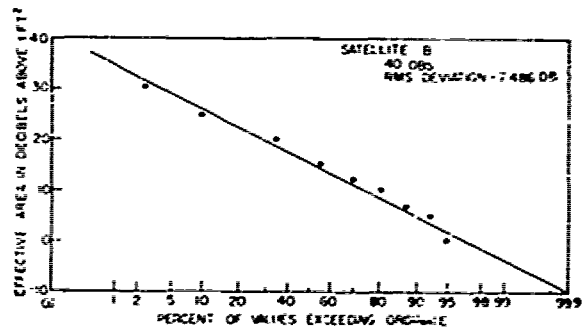(see Table A1 in Appendix A)

Fig. 2 - Agreement of typical cross sections cal-
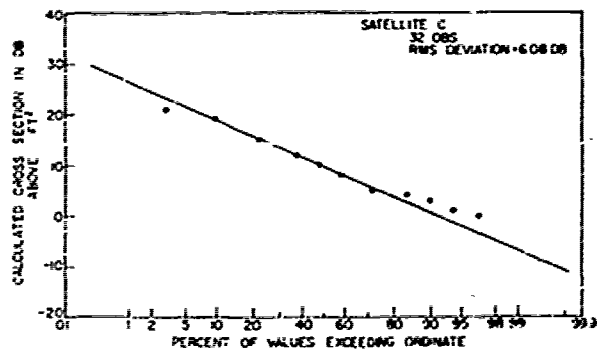culated for satellite B with lognormal distribution

Fig. 3 - Agreement of typical cross sections cal-
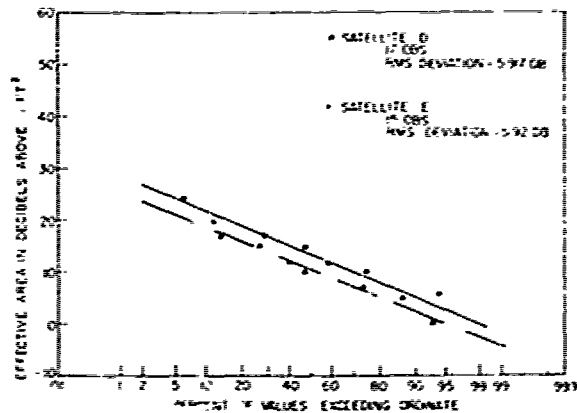culated for satellite C with lognormal distribution

Fig. 4 - Agreement of typical cross sections cal-
culated for satellites D and E with lognormal
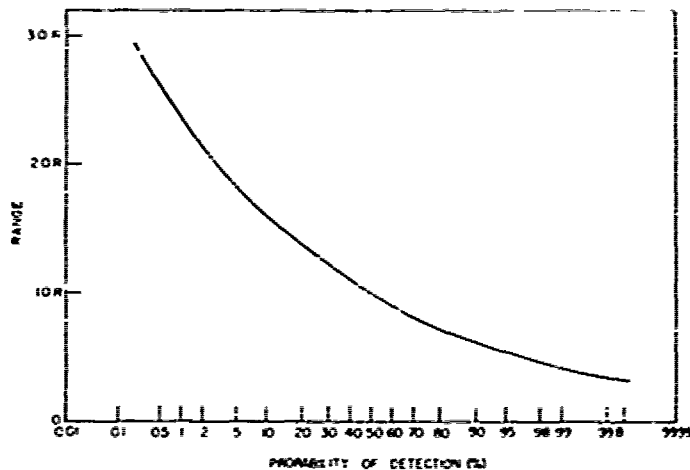distribution

Fig. 5 - Relation between range (in terms of nominal range)
and probability of detection

power of 18 db will cause the signal to exceed noise some 99.87% of the time.* If trans-
mitter power is fixed, then range must be reduced to improve the reliability. In this case,
because of the inverse fourth power effect of range, we must reduce range by 4.5 db, or
from the original range to 35% of that range. The relation between range and probability
of detection is shown in Fig. 5.

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*The 99.87% figure is obtained by integrating the probability density function of the normal
distribution. The integral may be found in tabular form on p. 1117 in "Reference Data for
Radio Engineers," fourth edition published by the International Telephone and Telegraph
Corporation, New York, 1956.

CONCLUSIONS

1. Even though the class of the variation in power returned from a satellite is unknown, knowledge of the form of the variation enables one to perform calculations that show the probability of interfering satellites.

2. When the data from many observations of several satellites are plotted the distribution most nearly fits the lognormal form with different standard deviations for the various satellites.

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APPENDIX A

SAMPLE CALCULATIONS OF REFLECTING CROSS SECTIONS

The known data on satellite passes are the zenith angles from the receiving stations. A program has been prepared for the NAREC computer to calculate ranges, height, transmitter zenith angle. etc., from the known receiver zenith angles, but for these calculations of cross section, the graphical solution using a vertical plot chart is sufficiently accurate (a sketch of a vertical plot is shown in Fig. A1). The ranges and the transmitter zenith angle are read off from the scaled construction. Figure A2 is a plot of antenna gain versus zenith angle for all stations. Thus, from the primary data we derive the parameters that appear in the equation

$$\frac{P_R}{P_T} = \frac{G_T \, G_R \, \sigma \, \lambda^2}{(4\pi)^3 \, R_T^2 \, R_R^2}.$$

Since the satellite is illuminated by two transmitters, the equation must be expanded to the form

$$\frac{P_R}{P_T} = \frac{G_R \, \sigma \, \lambda^2}{(4\pi)^3 \, R_R^2} \left(\frac{G_{T_1} P_{T_1}}{R_{T_1}^2} + \frac{G_{T_2} P_{T_2}}{R_{T_2}^2} \right).$$

The transmitter powers for the two stations are known and the received power is read off the Sanborn record of the agc voltage as shown in Fig. A3. Since range from transmitter and transmitter antenna gain are used twice in a triangulation. it is convenient to calculate the value K where

$$K = 5.39 \times 10^{-17} \left(\frac{G_{T_1} P_{T_1}}{R_{T_1}^2} + \frac{G_{T_2} P_{T_2}}{R_{T_2}^2} \right).$$

The constant puts all measurements in statute miles. Only in the case of distant large satellites such as Echo does the more remote transmitter contribute significantly to the target illumination. so that K in most cases reduces to

$$K = 5.39 \times 10^{-17} \frac{G_T \, P_T}{R_T^2}.$$

For each receiving station the reflecting cross sections are

$$\sigma = \frac{R_R^2 \, P_R}{K \, G_R} \text{ sq ft.}$$

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Calculations were made for five satellites (Table A1). For each satellite, the calculated areas were dispersed over a range of several decades. When the calculated areas are plotted in decibels referred to one square foot of area on cumulative probability paper, the agreement with lognormal distribution is good. These plots are presented in the main text as Figs. 1-4. The straight line on each figure has been drawn so as to show the exact lognormal distribution of a population with log mean area equal to that of the satellite sample and a standard deviation equal to the root-mean-square deviation of the satellite's calculated area in decibels above one square foot from the mean of those values.
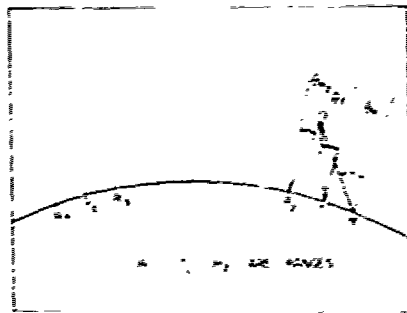
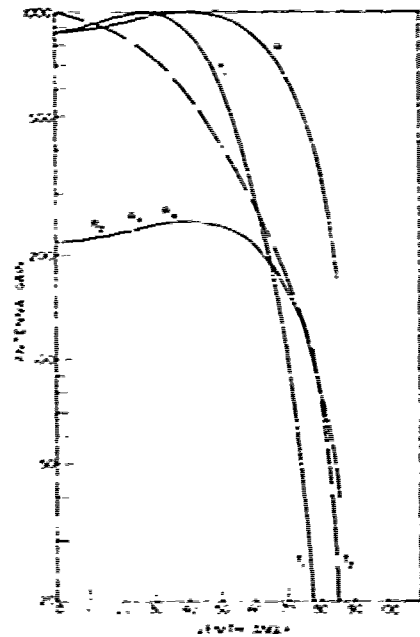Fig. A1 - Space Surveillance
System vertical plot

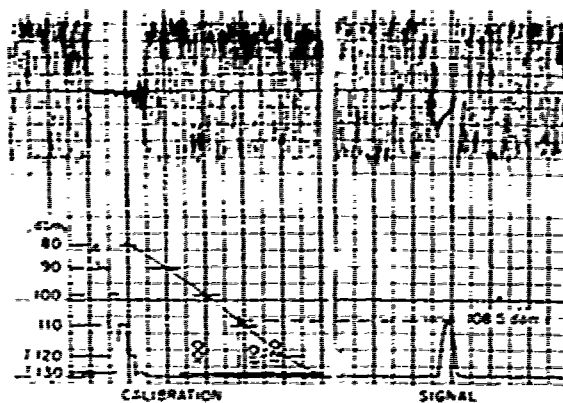Fig. A2 - Antenna patterns for all
stations (see Fig. A1)

Fig. A3 - Sample record of a satellite showing agc
voltage alongside a set of station calibrated power
levels

Table A1
Reflecting Cross Sections of Satellites as Calculated Using Received Signals
and Known Transmission

Cross Section (ft^2)						
Satellite A		Satellite B		Satellite C		Satellite D	Satellite E
5,982	9,430	15.6	4.95	19.8	27.0	78.5	58.6
757	267	111	872	11.2	25.6	71.5	0.319
4,030	3,890	365	61.2	8.06	1.34	6.84	4.7
14,400	5,080	134	218	4.3	146	14.2	54.3
6,850	596	128	25.4	3.78	46.8	32.1	2.03
780	519	79.8	151	7.66	12.2	1.24	27.6
736	9,350	98	163	22.2	1.02	36.7	15.9
2,009	6,100	62.2	15.4	2.21	50.8	27.3	37.5
13,120	3,180	162	24.4	32.9	2.42	35.2	81.4
2,100	7,240	0.736	87	50.5	10.5	276	36.8
4,520	1,010	120.6	66.2	9.76	99.6	21.4	2.1
4,610	4,210	16.8	279	6.13		3.54	4.75
262	4,850	31.9	0.87	25.3		5.02	7.92
5,000	1,840	3.43	6.56	98.8		4.0	12.7
106	6,660	2.43	23.6	3.42		53.2	14.0
493	20,900	5.22	34.6	2.62		4.02	
3,080	2,530	5.08	147	2.44		30.6	
13,700	4,950	15.4	23.9	0.443			
575	2,170	120	19.1	9.7			
3,110	8,020	36.3	12.2	2.78			
3,630		126		2.85			

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APPENDIX B

STATISTICAL TESTS

FITTING SAMPLE TO A KNOWN DISTRIBUTION

The sample distribution was compared with the theoretical lognormal distribution by using the χ^2 significance test as described in "Handbook of Probability and Statistics with Tables" by Burington and May,[*] page 178. The criterion for goodness of fit is that the cumulative deviation of the sample from the lognormal distribution be one that would be exceeded by chance at least 5% of the time. For the five samples of Appendix A, the deviations were such as to be exceeded by chance 15%, 45%, 74%, 36%, and 62% of the time. By this test, the agreement with the lognormal distribution is excellent.

CONFIDENCE IN DETERMINING MEAN CROSS SECTION

When a number of values of cross section of a satellite are computed, one feels intuitively that the mean is a more accurate value than any single computed value. This feeling is supported by the mathematics of statistics. One can determine by the Student's test, for example, that for a sample of size n from a population of known distribution, the sample mean will lie in a range bounded by $\pm t / \sqrt{n-1}$ standard deviations from the population mean with a confidence of p percent. The value of t is found in Table XII in "Handbook of Probability and Statistics with Tables" by Burington and May.[*] Where 41 values are available on satellite A, the test shows that the sample mean of 34.3 db above one square foot lies within $\pm 1.6 / \sqrt{40}$ standard deviation of the sample from the population mean with a confidence of 90%. The standard deviation is calculated as 5.35 db. Thus the 90% confidence interval extends from 32.92 db above one square foot to 35.78 db above one square foot. The 50% confidence interval extends from 33.77 db over one square to 34.93 db above one square foot. For satellite E, where 15 values make up the sample, the 90% confidence interval is bounded by $\pm 1.7 / \sqrt{14}$ standard deviations of the sample about the population mean. The interval is ± 2.2 db about the sample mean value of 10.5 db above one square foot. To summarize, a sample of 41 readings has a mean within ± 1.43 db of the population mean to a confidence of 90% and a sample of 15 readings yields a mean within ± 2.2 db of the population mean to the same confidence. If we assume that all of the satellites show the same sample standard deviation of 6 db, then a simple plot shows the relation between the number of readings and the size of the confidence limit (Fig. B1).

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[*] R. S. Burington and D. C. May, "Handbook of Probability and Statistics with Tables," Sandusky Ohio:Handbook Publishers, 1953.
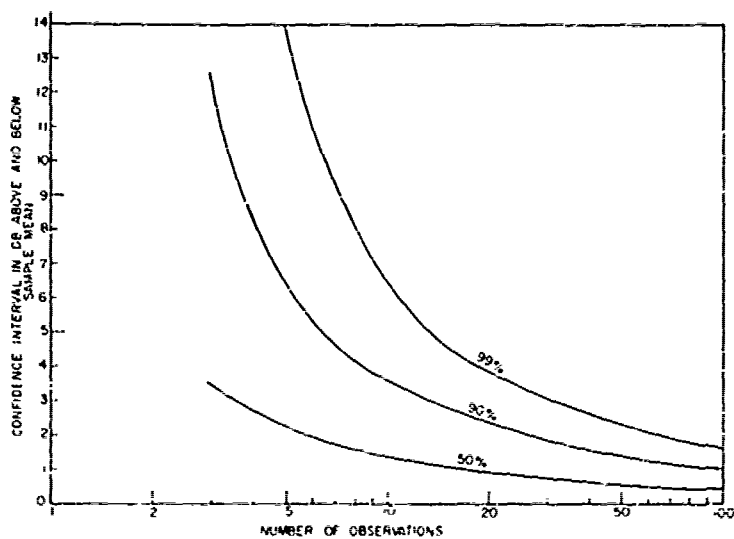
Fig. B1 - Confidence internal of satellite cross section about the sample mean; a sample standard deviation of 6 db is assumed

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